Isoscaling of the Fission Fragments with Langevin Equation*

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Langevin equation is used to simulate the fission process of ^{112}Sn + ^{112}Sn and ^{116}Sn + ^{116}Sn. The mass distribution of the fission fragments are given by assuming the process of symmetric fission or asymmetric fission with the Gaussian probability sampling. Isoscaling behavior has been observed from the analysis of fission fragments of both reactions and the isoscaling parameter α seems to be sensitive to the width of fission probability and the beam energy.

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Since the isoscaling law has been observed experimentally [1, 2, 3], many statistical models have successfully explained the isoscaling behavior. Isoscaling means that the ratio of isotope yields from two different reactions, 1 and 2, $R_{21}(N,Z) = Y_2(N,Z)/Y_1(N,Z)$, is found to exhibit an exponential relationship as a function of the neutron number N and proton number Z [1]

$$R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} = Cexp(\alpha N + \beta Z). \qquad (1)$$

where C, α and β are three parameters. In grand-canonical limit, $\alpha = \Delta\mu_n/T$ and $\beta = \Delta\mu_z/T$ where $\Delta\mu_n$ and $\Delta\mu_z$ are the differences between the neutron and proton chemical potentials for the two reactions, respectively. This behavior is attributed to the difference of two reaction systems with different isospin asymmetry. It is potential to probe the isospin dependent nuclear equation of state by the studies of isoscaling [4]. So far, the isoscaling behavior has been experimentally explored by various reaction mechanisms, ranging from the evaporation [1], fission [5, 6] and deep inelastic reaction at low energies to the projectile fragmentation [7, 8] and multi-fragmentation at intermediate energy [1, 9, 10]. While, the isoscaling phenomenon has been extensively examined in different theoretical frameworks, ranging from dynamical model, such as BUU model [9] and anti-symmetrical molecular dynamics model [11], to statistical models, such as the expansion emission source model, statistical multi-fragmentation model and the lattice gas model [2, 3, 12, 13, 14].

In this work, we present an analyse for the fragments from the fission which was simulated by Langevin equation. The isotopic or isotonic ratios of the different fragment yields from ^{116}Sn + ^{116}Sn and ^{112}Sn + ^{112}Sn system are presented and the features of the isoscaling behavior in fission dynamics are investigated.

The process of fission can be described in terms of collective motion using the transport theory [15, 16, 17, 18]. The dynamics of the collective degrees of freedom is typically described using the Langevin or Fokker-Planck equation. In this Letter, we deal with a Combine Dynamical and Statistical Model (CDSM) which is a combination of a dynamical Langevin equation and a statistical model to describe the fission process of heavy ion reaction. This model is an overdamped Langevin equation coupled with a Monte Carlo procedure allowing for the discrete emission of light particles. It switches over to statistical model when the dynamical description reaches a quasi-stationary regime. We first specify the entrance channel through which a compound nucleus is formed, ie. the target and projectile is complete fusion.

In this work the total initial excitation energy E^*_{tot} is given by $E^*_{tot} = E_{lab}A_T/(A_T + A_P) + Q$ where A_T and A_P represents the mass of target and projectile, respectively, and Q is the fusion Q-value calculated by $Q = M_T + M_P - M^{LD}_{CN}$. M_T and M_P is the mass of projectile and target come from experimental data, respectively. If it is unavailable, it is calculated by macroscopic-microscopic model [19]. M^{LD}_{CN} is the mass of the compound nucleus which is calculated from the liquid-drop model.

The dynamical part of CDSM model is described by Langevin equation which is driven by the free energy F. F is related to the level density parameter $a(q)$ [20]

$$F(q,T) = V(q) - a(q)T^2 \qquad (2)$$

in the Fermi gas model, where $V(q)$ is fission potential.

The overdamped Langevin equation reads

$$\frac{dq}{dt} = -\frac{1}{M\beta(q)}\left(\frac{\partial F(q,T)_T}{\partial q}\right) + \sqrt{D(q)}\Gamma(t), \qquad (3)$$

where q is the dimensionless fission coordinate defined as half of the distance between the centers of masses of the future fission fragments. $\Gamma(t)$ is a time-dependent stochastic variable with Gaussian distribution. Its average and correlation function is written as

$$\begin{aligned} &< \Gamma(t) >= 0, \\ &< \Gamma(t)\Gamma(t') >= 2\delta_\varepsilon(t - t'). \end{aligned} \qquad (4)$$

The fluctuation strength coefficient $D(q)$ can be expressed according to the fluctuation-dissipation theorem:

$$D(q) = \frac{T}{M\beta_0(q)}, \tag{5}$$

where M is the total mass and $\beta_0(q)$ is the reduced friction parameter which is the only parameter of this model.

The potential energy $V(A, Z, L, q)$ is obtained from the finite-range liquid drop model [21]

$$V(A, Z, L, q) = a_2[1 - k(\frac{N-Z}{A})^2]A^{2/3}[B_s(q) - 1] + c_3\frac{Z^2}{A^{1/3}}[B_c(q) - 1] + c_r L^2 A^{-5/3} B_r(q), \tag{6}$$

where $B_s(q)$, $B_c(q)$ and $B_r(q)$ means surface, Coulomb and rotational energy terms, respectively, which depends on the deformation coordinate q. a_2, c_3, k and c_r are parameters not related to q. In our calculation we take them according to Ref. [15].

We use c and h [22] to describe the shape of nucleus,

$$\rho^2(z) = (1 - \frac{z^2}{c_0^2})((\frac{1}{c^3} - \frac{b_0}{5})c_0^2 + B_{sh}(c, h)z^2), \tag{7}$$

where

$$c_0 = cR, \quad R = 1.16A^{1/3}. \tag{8}$$

The nuclear shape function $B_{sh}(c, h)$ and the collective fission coordinate $q(c, h)$ of mass number A is expressed as

$$B_{sh}(c, h) = 2h + \frac{c-1}{2},$$
$$q(c, h) = \frac{3}{8}c(1 + \frac{2}{15}B_{sh}(c, h)c^3). \tag{9}$$

The fission process of Langevin equation is propagated using an interpretation of Smoluchowski [23]. In our calculation we adopt one-body dissipation (OBD) friction form factor $\beta_0(q)$ [24] which is calculated with one-body dissipation with a reduction of wall term. Here we use an analytical fit formula which was developed in Ref. [25]

$$\beta_{OBD}(q) = \{ \begin{array}{ll} 15/q^{0.43} + 1 - 10.5q^{0.9} + q^2 & \text{if } q > 0.38 \\ 32 - 32.21q & \text{if } q < 0.38 \end{array}$$

In the dynamical part of the model the emission of light particles (n, α, p, d) and giant dipole γ are calculated at each Langevin time step τ, the widths for particle and giant dipole γ decay are given by the parametrization of Blann [26] and Lynn [27], respectively.

Within the framework of Langevin simulation we chose 200,000 fission events which happen on dynamic channel (we give up the events which happen in statistic part of CDSM model) and chose a Gaussian distribution random number as the mass asymmetry parameter $\alpha_0 = (A_1 - A_2)/(A_1 + A_2)$, which is defined as the ratio of the volumes of two parts of the nucleus obtained when it reaches the scission point. When $\alpha_0 = 0$ means symmetrical fission. It is taken from a Gaussian distribution random number from -1 to 1 with the mean is 0. A_1 and A_2 is the mass of the two fission fragments, respectively. In this work we assume the fission fragments have the same N/Z as the initial system and then Z_1 or Z_2 of fission fragments can be deduced from A_1 or A_2. This assumption is similar to the case of of deep inelastic heavy ion collisions at low energies, where the isospin degree of freedom has been found to first reach equilibrium [28].

From a practical point of view, the isoscaling occurs when two mass distributions for a given Z from two processes with different isospin are Gaussian distributions with the same width but different mean mass. Basically, the isotopic distribution can be described by

$$Y(N, Z) = f(Z)exp[-\frac{(N - N_Z)^2}{2\sigma_Z^2}], \tag{10}$$

where N_Z is the centroid of the distribution and σ_Z^2 describes the variance of the distribution for each element of charge Z. This leads to an exponential behavior of the ratio R_{21} if the quadratic term in N_Z is neglected,

$$ln(R_{21}) \sim \frac{[(N_Z)_2 - (N_Z)_1]N}{\sigma_Z^2}. \tag{11}$$

Note that Eq.(11) requires the values for the σ_Z^2 to be approximately the same for both reactions, which is a necessary condition for isoscaling. Indeed, we observed this case in our simulations for Sn + Sn collisions. In the Langevin equation, σ_Z^2 mainly depends on the physical conditions reached, such as the temperature, the density and the friction parameter etc. Considering that $R'_{21} = exp(\alpha N)$ for a given Z, $\alpha \sim \frac{[(N_Z)_2 - (N_Z)_1]}{\sigma_Z^2}$. Usually σ_Z^2 can be considered to be proportional to temperature T of the fragments, in this way

$$\alpha \sim \frac{[(N_Z)_2 - (N_Z)_1]}{T}, \tag{12}$$

where $[(N_Z)_2 - (N_Z)_1]$ can be understood as a term of the chemical potential difference between two reactions.

Eq.(1) can be written as $lnR_{21} = C_Z + \alpha N$, where $C_Z = lnC + \beta Z$, if we plot R_{21} as function of N, on a natural logarithmic plot, the ratio follows along a straight line. In Fig. 1 this behavior is observed in Langevin simulation. From the figure, the relationship between α and the charge number Z of the fission fragments can be deduced. In order to investigate the effect of the width of Gaussian probability distribution on the isoscaling behavior, we change the widths of the Gaussian distribution of mass asymmetry parameter α_0, namely $\sigma_{\alpha_0} = 0.04$, 0.06, 0.08 and 0.20, with the random number from -1 to 1 and the mean of 0. Fig.2 shows the isoscaling parameter α as a function of Z in the conditions of different

σ_{α_0}. From this figure, we know in the low σ_{α_0}, i.e., the fission fragments are overwhelmingly dominated by the symmetric fission, α increases with Z. This means that the isospin effect becomes stronger with the increasing of Z. In a recent analyse of Friedman [5] with a simple liquid-drop model, a systematic increase of the isoscaling parameter α with the proton number of the fragment element is predicted. In our simulation, this kind of increase of α with Z stems from the dominated symmetric fission mechanism. While, in the another extreme case from the Fig.2, i.e. with the larger σ_{α_0}, α shows a contrary trend with Z, i.e., it drops with Z. In this case, it seems that there exists stronger isospin effect for the fragments with lower Z. In a middle case, the rising branch and falling branch competes with each other, the mediate isoscaling behavior appears and a minimum of α parameter occurs around the symmetric fission point. We note that the fission data of ^{238,233}U targets induced by 14 MeV neutrons reveals the backbending behavior of the isoscaling parameter α around the symmetric fission point [6] as stated above. In their study, they interpreted this originates from the temperature difference of fission fragments since the isoscaling parameter is typically, within the grand-canonical approximation, considered inversely proportional to the temperature ($\alpha = \Delta\mu_n/\mathrm{T}$) as stated above. It is, however, not *a priori* obvious why such a grand-canonical formula can be applied to fission. In our case, this kind of backbending of isoscaling parameter apparently stems from the moderate width of the fission probability of the fissioning nucleus as shown in Fig.2. Of course, we did not exclude the change of temperature of the fission fragments due to the change of the width of the fission probability. Overall speaking, we find that the isoscaling behavior is sensitive to the width of the fission probability distribution.

In addition, the simulations are systematically done in different beam energies. The values of α are extracted as a function of beam energy for the fragments Z = 44-52 as shown in Fig.3. It shows that α decreases as the beam energy increasing which means that the isospin effect fades away with the increasing of E_{lab}. This behavior is similar to that α drops with the temperature in the statistical models as well as experiments [3, 12, 29, 30].

In summary, we applied Langevin model to investigate the isoscaling behavior in the dynamical process of compound nuclear fission. In order to treat the fission fragments, we assume that the mass asymmetry parameter of the fissioning nucleus is taken from a random number of Gaussian distribution whose width is σ_{α_0}. The simulation illustrates that the yield ratios of fission fragments in the dynamical fission of $^{116}Sn+^{116}Sn$ and $^{112}Sn+^{112}Sn$ reaction system shows the isoscaling behavior. More interestingly, we found that the isoscaling parameter α is sensitive strongly to the Gaussian width σ_{α_0} of the mass asymmetry parameter. When σ_{α_0} is small, i.e. the fission is almost symmetrical, α increases with the atomic number of fission fragments, which is similar to the theoretical prediction of a simple liquid-drop model [5]. In contrary, when σ_{α_0} is large, for instance, $\sigma_{\alpha_0} = 0.20$, α drops with Z of fission fragments. However, in the intermediate values of σ_{α_0}, α shows a backbending with Z of fission fragments, which is similar to the observation of the ^{238,233}U fission data induced by 14 MeV neutrons [6]. In addition, it is found that α drops with the beam energy of the projectile, reflecting the temperature dependence of isoscaling parameter. In general, the isoscaling analysis of the fission data appears to be a sensitive tool to investigate the fission dynamics.

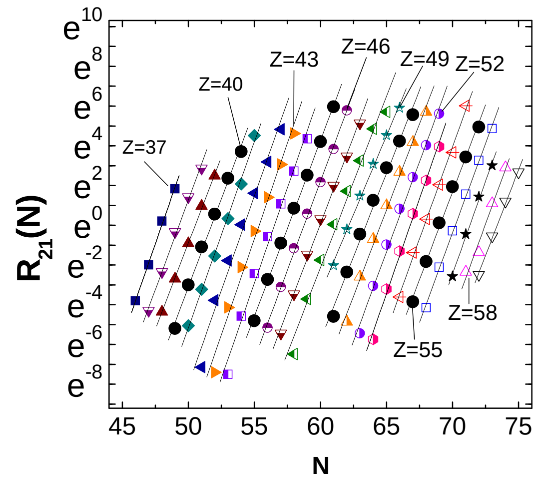

FIG. 1: The yield ratio of the fission fragments between $^{116}Sn+^{116}Sn$ and $^{112}Sn+^{112}Sn$ in the Langevin model with $\sigma_{\alpha_0} = 0.06$ and E/A = 8.4 MeV. Different symbols from left to right represent the calculated results for the isotopes from Z = 37 to 59. The lines represent exponential fits to guide the eye.

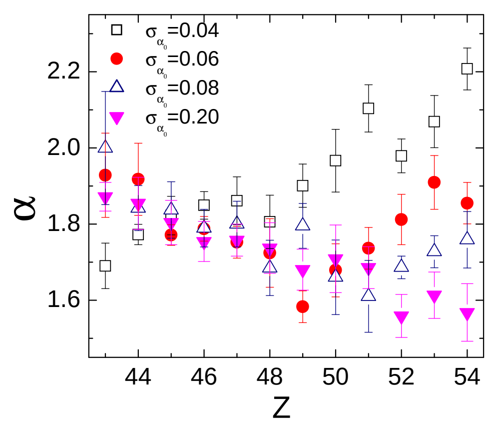

FIG. 2: The scaling parameter α as a function of Z in the different width (σ_{α_0}) of the mass asymmetry parameter α_0 with Gaussian distribution for fission.

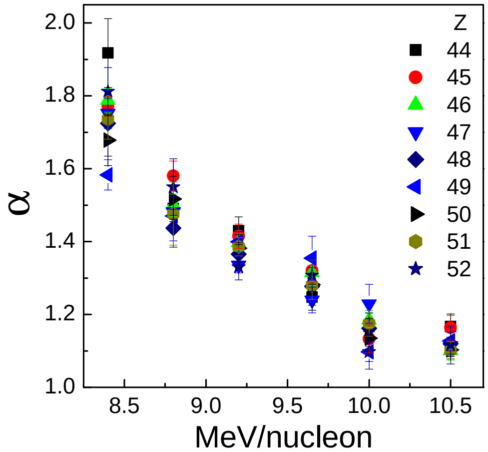

FIG. 3: α as a function of beam energy for the fragments Z = 44-52. The width of the Gaussian probability σ_{α_0} is 0.06.

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